Noise in Communication Systems

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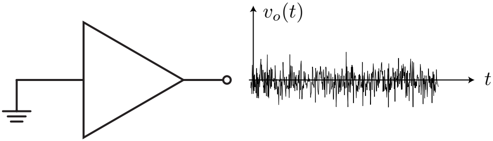
Introduction to Noise



- All electronic amplifiers generate noise. This noise originates from the random thermal motion of carriers and the discreteness of charge.
- Noise signals are random and must be treated by statistical means. Even though we cannot predict the actual noise waveform, we can predict the statistics such as the mean (average) and variance.

Noise Power

- The average value of the noise waveform is zero

$$\overline{v_n}(t) =< v_n(t) >= \frac{1}{T}\int_T v_n(t)dt = 0$$

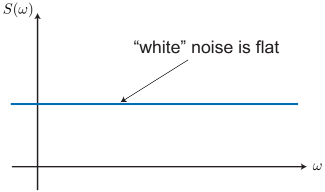
- The mean is also zero if we freeze time and take an infinite number of samples from identical amplifiers.
- The variance, though, is non-zero. Equivalently, we may say that the signal power is non-zero

$$\overline{v_n(t)^2} = \frac{1}{T}\int_T v_n^2(t)dt \neq 0$$

- The RMS (root-mean-square) voltage is given by

$$v_{n,rms} = \sqrt{\overline{v_n(t)^2}}$$

Power Spectrum of Noise



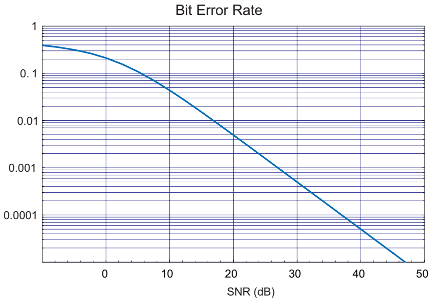
The power spectrum of the noise shows the concentration of noise power at any given frequency. Many noise sources are "white" in that the spectrum is flat (up to extremely high frequencies)

- In such cases the noise waveform is totally unpredictable as a function of time. In other words, there is absolutely no correlation between the noise waveform at time t_1 and some later time $t_1 + \delta$, no matter how small we make δ.

Noise in Communication Systems

- Noise is an ever present part of all systems. Any receiver must contend with noise and the resulting degradation in the quality of the transmitted and received signal.
- In analog systems, noise deteriorates the quality of the received signal, e.g. the appearance of "snow" on the TV screen, or "static" sounds during an audio transmission.
- In digital communication systems, noise degrades the throughput because it requires retransmission of data packets or extra coding to recover the data in the presence of errors.

BER Plot



- It's typical to plot the Bit-Error-Rate (BER) in a digital communication system.
- This shows the average rate of errors for a given signal-to-noise-ratio (SNR)

SNR

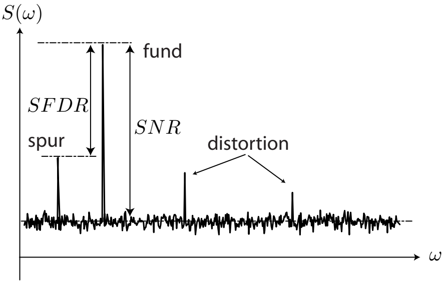
- In general, then, we strive to maximize the signal to noise ratio in a communication system. If we receive a signal with average power P_{sig}, and the average noise power level is P_{noise}, then the SNR is simply

$$SNR = \frac{S}{N}$$

$$SNR(\mathrm{dB}) = 10 \cdot \log \frac{P_{sig}}{P_{noise}}$$

- We distinguish between random noise and "noise" due to interferers or distortion generated by the amplifier

Spurious Free Dynamic Range



- The spurious free dynamic range *SFDR* measures the available dynamic range of a signal at a particular point in a system. For instance, in an amplifier the largest signal determines the distortion "noise" floor and the noise properties of the amplifier determine the "noise floor"

Noise Figure

- The *Noise Figure* (NF) of an amplifier is a block (e.g. an amplifier) is a measure of the degradation of the SNR

$$F = \frac{SNR_i}{SNR_o}$$

$$NF = 10 \cdot \log(F) \ (\mathrm{dB})$$

- The noise figure is measured (or calculated) by specifying a standard input noise level through the source resistance R_s and the temperature
- For RF communication systems, this is usually specified as $R_s = 50\Omega$ and $T = 293^\circ K$.

- Suppose an amplifier has a gain G and apply the definition of NF

$$SNR_i = \frac{P_{sig}}{N_s}$$

$$SNR_o = \frac{GP_{sig}}{GN_s + N_{amp,o}}$$

- The term $N_{amp,o}$ is the total output noise due to the amplifier in absence of any input noise.

$$SNR_o = \frac{P_{sig}}{N_s + \frac{N_{amp,o}}{G}}$$

Input Referred Noise (I)

- Let $N_{amp,i}$ denote the total input referred noise of the amplifier

$$SNR_o = \frac{P_{sig}}{N_s + N_{amp,i}}$$

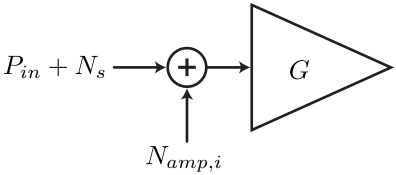
- The noise figure is therefore

$$F = \frac{SNR_i}{SNR_o} = \frac{\cancel{P_{sig}}}{N_s} \times \frac{N_s + N_{amp,i}}{\cancel{P_{sig}}}$$

$$F = 1 + \frac{N_{amp,i}}{N_s} \geq 1$$

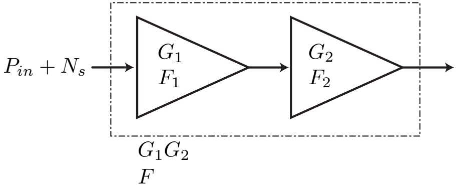
- All amplifiers have a noise figure ≥ 1. Any real system degrades the SNR since all circuit blocks add additional noise.

Input Referred Noise (II)



- The amount of noise added by the amplifier is normalized to the incoming noise N_s in the calculation of F. For RF systems, this is the noise of a 50Ω source at $293^\circ\mathrm{K}$.
- Since any amplification degrades the *SNR*, why do any amplification at all? Because often the incoming signal is too weak to be detected without amplification.

Noise Figure of Cascaded Blocks



- If two blocks are cascaded, we would like to derive the noise figure of the total system.
- Assume the blocks are impedance matched properly to result in a gain $G = G_1 G_2$. For each amplifier in cascade, we have

$$F_i = 1 + \frac{N_{amp,i}}{N_s}$$

Total Input Noise for Cascade

- By definition, the noise added by each amplifier to the input is given by

$$N_{amp,i} = N_s(F-1)$$

- where N_s represents some standard input noise. If we now input refer all the noise in the system we have

$$N'_{amp,i} = N_s(F_1-1) + \frac{N_s(F_2-1)}{G_1}$$

- Which gives us the total noise figure of the amplifier

$$F = 1 + \frac{N'_{amp,i}}{N_s} = 1 + (F_1-1) + \frac{F_2-1}{G_1} = F_1 + \frac{F_2-1}{G_1}$$

General Cascade Formula

- Apply the formula to the last two blocks

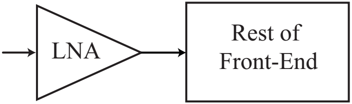
$$F_{23} = F_2 + \frac{F_3 - 1}{G_2}$$

$$\begin{aligned} F &= F_1 + \frac{F_{23} - 1}{G_1} \\ &= F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} \end{aligned}$$

- The general equation is written by inspection

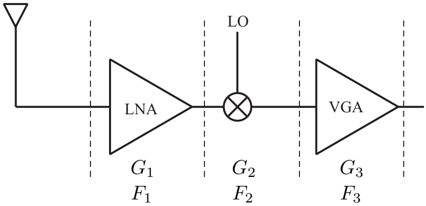
$$= F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} + \cdots$$

Cascade Formula Interpretation



- We see that in a cascade, the noise contribution of each successive stage is smaller and smaller.
- The noise of the *first* stage is the most important. Thus, every communication system employs a *low noise amplifier* (LNA) at the front to relax the noise requirements
- A typical LNA might have a $G = 20\,\mathrm{dB}$ of gain and a noise figure $NF < 1.5\,\mathrm{dB}$. The noise figure depends on the application.

NF Cascade Example



- The LNA has $G = 15\,\mathrm{dB}$ and $NF = 1.5\,\mathrm{dB}$. The mixer has a conversion gain of $G = 10\,\mathrm{dB}$ and $NF = 10\,\mathrm{dB}$. The IF amplifier has $G = 70\,\mathrm{dB}$ and $NF = 20\,\mathrm{dB}$.
- Even though the blocks operate at different frequencies, we can still apply the cascade formula if the blocks are impedance matched

$$F = 1.413 + \frac{10-1}{60} + \frac{100-1}{60 \cdot 10} = 2.4\,\mathrm{dB}$$

Minimum Detectable Signal

- Say a system requires an SNR of $10\,\mathrm{dB}$ for proper detection with a minimum voltage amplitude of $1\mathrm{mV}$. If a front-end with sufficient gain has $NF = 10\,\mathrm{dB}$, let's compute the minimum input power that can support communication:

$$SNR_o = \frac{SNR_i}{F} = \frac{\frac{P_{min}}{N_s}}{F} > 10$$

or

$$P_{in} > 10 \cdot F \cdot N_s = 10 \cdot F \cdot kTB$$

- we see that the answer depends on the bandwidth B.

$$P_{in} = 10\,\mathrm{dB} + NF - 174\,\mathrm{dBm} + 10 \cdot \log B$$

Minimum Signal (cont)

- For wireless data, $B \sim 10\mathrm{MHz}$:

$$P_{in} = 10\,\mathrm{dB} + 10\,\mathrm{dB} - 174\,\mathrm{dB} + 70\,\mathrm{dB} = -84\,\mathrm{dBm}$$

- We see that the noise figure has a dB for dB impact on the minimum detectable input signal. Since the received power drops $> 20\,\mathrm{dB}$ per decade of distance, a few dB improved NF may dramatically improve the coverage area of a communication link.
- Otherwise the transmitter has to boost the TX power, which requires excess power consumption due to the efficiency η of the transmitter.